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THE WELFARE APPROACH TO MEASURING INEQUALITY

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Recently, sociologists have expressed a renewed interest in the theoretical and empirical study of inequality, its determinants, and its effects. Recent studies include Gartrell (1977), Rubinson and Quinlan (1977), Blau (1977), Jencks and others (1972), and Chase-Dunn (1975).[1] In such studies the analysts usually choose a single index to measure inequality, such as the coefficient of variation or the Gini coefficient, and then use it to analyze their data. With the exception of Blau, few have made an explicit attempt to define the concept of inequality or to justify the chosen index as an appropriate measure of inequality. However, choosing a single index from the available ones implies that inequality is a unidimensional concept and that the chosen index is a valid measure of it.

But it is not necessarily the case that different measures of inequality will correlate highly with the concept and with each other and that they will therefore rank distributions in the same order. Different measures may yield different results, and the differences may be considerable. We demonstrate this by analyzing the Kuznets data (1963) on the distribution of individual income for 12 countries in about 1950. Table 1 presents rank-order correlations (Kendall's tau) among four commonly used measures of inequality applied to data (Tables 2 to 4): the coefficient of variation (CV), the Gini coefficient (GC), the standard deviation of the logarithm (SDL), and the mean relative deviation (MRD). Formulas for these measures are given in the appendix.

The first three measures are commonly used to measure income inequality; the mean relative deviation is used for this purpose and for measuring degree of segregation.[2] The correlations

[1] The recent paper by Allison (1978) discusses, with a different emphasis, some of the issues explored in this chapter. Except for this note, we make no reference to it, mainly because we have had too little time to consider its content critically.

[2] In this context the mean relative deviation is known as the index of dissimilarity. Duncan and Duncan (1955) show that measuring segregation is structurally similar to measuring economic inequality. (See also Winship, 1978.) Our comments about measures of inequality therefore pertain also to measures of segregation. Although measures of inequality have been applied to many problems outside economics (for example, education; see Blau, 1977), we limit our discussion to the problem of measuring economic inequality. See Agresti and Agresti (1977) for a discussion related to measuring inequality in the distribution of a nominal variable.

TABLE 1
Kendall Rank-Order Correlations (Tau) Between Different Measures of Inequality

Measure	CV	MRD	GC	SDL
Coefficient of variation	1.000	0.727	0.697	0.152
Mean relative deviation		1.000	0.909	0.424
Gini coefficient			1.000	0.454
Standard deviation of logarithm				1.000

of the standard deviation of the logarithm of income with each of the other measures are the lowest—0.152, 0.424, and 0.454. The correlations between the coefficient of variation and the mean relative deviation and Gini coefficient are moderately large. Even the correlation of the mean relative deviation and Gini coefficient is not as high as one might expect from the similarity of their definitions.

For an example of the point that different measures may yield inconsistent rankings, consider India and Sweden: India is ranked ninth, eleventh, eleventh, and third by the CV, the MRD, the GC, and the SDL, respectively. Sweden is ranked sixth, fourth, fourth, and eleventh by each of these respective measures.

TABLE 2
Percentage of Total Income Received by Ranked Cohorts of Population

Country	Year	0–20	20–40	40–60	60–80	80–90	90–95	95–100
India	1950	7.82	9.22	11.4	16	12.4	9.62	33.5
Ceylon	1952–1953	5.1	9.3	13.3	18.4	13.3	9.6	31
Mexico	1957	4.4	6.9	9.0	17.4	14.7	9.7	37
Barbados	1951–1952	3.6	9.3	14.2	21.3	17.4	11.9	22.3
Puerto Rico	1953	5.6	9.8	14.9	19.9	16.9	9.5	23.4
Italy	1948	6.09	10.5	14.6	20.4	14.4	9.99	24.1
Great Britain	1951–1952	5.4	11.3	16.6	22.2	14.3	9.3	20.9
West Germany	1950	4	8.5	16.5	23	14	10.4	23.6
Netherlands	1950	4.2	9.6	15.7	21.5	14	10.4	24.6
Denmark	1952	3.4	10.3	15.8	23.5	16.3	10.6	20.1
Sweden	1948	3.2	9.6	16.3	24.3	16.3	10.2	20.1
United States	1950	4.8	11	16.2	22.3	15.4	9.9	20.4
Average		4.8	9.61	14.6	10.9	15.0	10.1	25.1

SOURCE: Kuznets (1963, table 3).

TABLE 3
Percentage of Total Income Received by Poorest X Percent of Population

Country	20	40	60	80	90	95	100
India	7.82	17	28.5	44.5	56.9	66.5	100
Ceylon	5.1	14.4	27.7	46.1	59.4	69	100
Mexico	4.4	11.3	21.2	38.6	53.3	63	100
Barbados	3.6	12.9	27.1	48.4	65.8	77.7	100
Puerto Rico	5.6	15.4	30.3	50.2	67.1	76.6	100
Italy	6.09	16.6	31.2	51.5	65.9	75.9	100
Great Britain	5.4	16.7	33.3	55.5	69.8	79.1	100
West Germany	4	12.5	29	52	66	76.4	100
Netherlands	4.2	13.8	29.5	51	65	75.4	100
Denmark	3.4	13.7	29.5	53	69.3	79.9	100
Sweden	3.2	12.8	29.1	53.4	69.7	79.9	100
United States	4.8	15.8	32	54.3	69.7	79.6	100
Average	4.80	14.41	29.03	49.88	64.83	74.92	100

SOURCE: Kuznets (1963, table 3).

TABLE 4
Indices of Inequality for Data in Table 3

					Atkinson's Measure		
Country	CV	MRD	GC	SDL	$e = 0.5$	$e = 1.0$	$e = 2.0$
India	1.359	0.355	0.451	0.338	0.188	0.295	0.397
Ceylon	1.264	0.339	0.465	0.373	0.188	0.311	0.457
Mexico	1.541	0.414	0.544	0.428	0.251	0.401	0.550
Barbados	0.979	0.329	0.454	0.418	0.170	0.315	0.524
Puerto Rico	0.966	0.298	0.417	0.350	0.144	0.256	0.408
Italy	0.976	0.288	0.405	0.334	0.138	0.242	0.380
Great Britain	0.845	0.267	0.378	0.333	0.122	0.224	0.384
West Germany	0.985	0.310	0.437	0.399	0.163	0.299	0.498
Netherlands	1.016	0.305	0.434	0.387	0.161	0.290	0.478
Denmark	0.871	0.305	0.421	0.410	0.151	0.292	0.520
Sweden	0.872	0.309	0.425	0.422	0.156	0.303	0.540
United States	0.847	0.280	0.393	0.354	0.130	0.242	0.420

How do differences between indices affect conclusions about the relationship of inequality to other variables? Kuznets (1963) investigated the association between income inequality and level of economic development in different countries. He noted (p. 17) that the SDL is especially sensitive to the percentage of national income received by the poor, while the GC is more sensitive to the share received by the rich,[3] and used this difference to explain why these two measures lead to different conclusions about the relationship between inequality and economic development. He went on to show that the developed countries (Tables 2 to 4) have lower GCs (indicating less inequality) than the underdeveloped countries, but about equal SDLs.[4] Kuznets did not conclude that the latter index does not measure inequality, but rather that level of economic development is not associated with inequality at the bottom of the distribution while it is strongly related to inequality in the top of the distribution.

What are the implications of these results? One implication is that different indices may be measuring different aspects of inequality; another is that different measures may not be equally valid indicators of the same concept. But whether inequality is regarded as unidimensional or multidimensional, we need criteria for evaluating the validity of devices purporting to measure it. Similarly, we need criteria for determining when one distribution is more unequal than another. Such criteria must, we believe, be based on a prior theoretical conceptualization of inequality.

Since the late 1960s, a considerable literature has developed in economics that addresses these issues: Aigner and Heins (1967), Kolm (1969), Kondor (1975), Sen (1973), and many articles in *The Journal of Economic Theory*. We have termed it "the welfare approach to measuring inequality." The roots of this work are found in Lorenz (1905), Pigou (1912, 1920), and especially Dalton (1920). Although much of the important work was done in the early 1970s, sociologists seem to be unaware of it. Economists have not only suggested a number of new and important measures of inequality, but they have also clarified many conceptual issues involved in

[3] The Gini coefficient is measured on the untransformed income scale; the standard deviation of the logarithm is based on squared differences on a log income scale that magnifies differences at the bottom of the scale while reducing differences at the top.

[4] When we used the Mann-Whitney test to examine ordinal differences in inequality between the developed and underdeveloped countries, we found that three measures indicated significant differences (at the 0.05 level) and one did not: Gini coefficient ($P = 0.015$); mean relative deviation ($P = 0.015$); coefficient of variation ($P = 0.024$); and standard deviation of the logarithm ($P = 0.378$).

determining whether distributions are identical in their inequalities. Although the new measures are embedded in economic theory, we shall show that at least one family of measures (Atkinson's) can be interpreted within a more general framework.

All measures of inequality imply judgement about the definition of inequality, about how to compare the inequality of various income distributions, and about what type of change will have the greatest effect on inequality. The Atkinson measures force the researcher to make these judgements explicit. These judgements will affect substantive findings and conclusions drawn from them.

The theory as developed in economics has two components: a basic theory that is independent of welfare economics and an elaboration of that theory which relies heavily on welfare economics. The basic theory enjoys considerable consensus among economists. We suspect that sociologists will find little that is objectionable and many ideas that are already familiar. Although well developed, the basic theory is incomplete in that it allows us to determine only in certain special cases whether one distribution is more equally distributed than another.

THE BASIC THEORY

We assume that all inequality measures share a number of formal properties. First, they are zero when incomes are distributed equally and positive otherwise. Second, they are impartial in that they do not depend on who possesses what income. The four traditional measures discussed above all exhibit these properties.

The basic theory has three axioms or assumptions. While these axioms do not constitute a complete definition of inequality, we expect most definitions to be consistent with them.

Axiom 1: Principle of Transfers

Basic to any notion of inequality is the idea that inequality is reduced if we transfer income from a richer person to a poorer person. It is understood that the transfer should not be so large that the receiver becomes richer than the donor. This concept has become known as the Pigou-Dalton principle of transfers (hereafter referred to as the transfers principle).

The transfers principle allows us to compare distributions involving the same number of persons and the same mean income. If one distribution of income (A) can be made to match another (B) by transferring money from rich to poor (in A), then we suppose distribution A is less equal than B. In order to make comparisons between populations with differing numbers of people and differing mean incomes, however, we require two additional axioms.

Axiom 2: Population Symmetry

If two populations are equal in size and identically distributed in income, then income inequality is identical in each. Moreover, it seems reasonable to assume that inequality in the combined population is the same as inequality in each of the two separate populations. Sen (1973) has labeled this concept the symmetry axiom for populations. The symmetry axiom allows us to compare distributions for groups of unequal size but with the same mean income. Given two populations with differing numbers of people (m and n), we need only add the first population n times to itself and the second population m times to itself to obtain two populations with the same total number of people and same mean income. We can then compare one population with the other by using the transfers principle.

Axiom 3: Scale Invariance

The symmetry axiom for populations allows us to deal with populations of different sizes. But how are we to treat populations with different income means? The usual assumption is that if we increase every individual's income by the same proportion then income inequality will remain unchanged. In other words, the size of the pie to be divided has no bearing on the *degree* of inequality—it is only the *relative* share each person receives that is important in determining inequality.

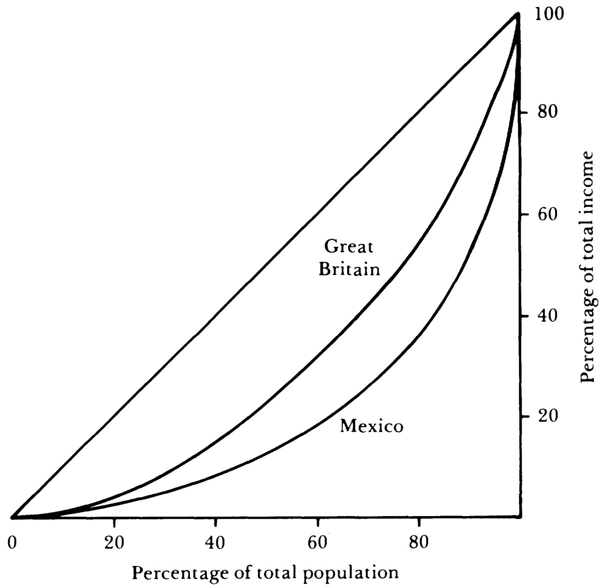
Not all students of the problem find this axiom acceptable. Dalton (1920), for instance, believed that adding the same amount of income to each person's income decreases inequality, whereas proportionate additions increase it. Research to date has not produced a satisfactory conclusion about the acceptability of this axiom. (See Kolm 1976a and 1976b.) For the present we tentatively accept the axiom (which we shall call the scale invariance axiom) and apply it in our work.[5]

Lorenz Criterion

All three axioms are intimately related to the Lorenz criterion. In applying this criterion the analyst orders people from the poorest to the richest. The Lorenz curve is the graph of the percentage of total income (the *Y* coordinate) possessed by the *X* poorest percent of the population. Figure 1 shows the Lorenz curves from the Kuznets (1963) data for Great Britain and Mexico.

[5] Increasing everyone's income proportionately may leave income inequality unchanged but increase overall inequality. This will occur if income is more unequally distributed than other resources and income inequality is a large part of overall inequality. By increasing everyone's income we increase the importance of income in overall inequality and thus increase overall inequality. Income inequality, however, is the same. A resulting implication is that decreasing income inequality may be considered objectionable if it produces an increase in overall social inequality.

Figure 1. Lorenz curves for Great Britain and Mexico.



The Lorenz criterion states that income in *A* is more equally distributed than income in *B* if the Lorenz curve for *A* is nowhere below and somewhere above the Lorenz curve for *B*. Thus in Figure 1 income is more equally distributed in Great Britain than it is in Mexico. One justification for the Lorenz criterion is that in the distribution with the higher curve, the poorest *X* percent of the population always has an equal if not a larger share of the total income than the poorest *X* percent of the population in the other distribution for all *X* between zero and 100 percent.

The Lorenz criterion has a special relationship with the three axioms cited earlier. If we have two populations of the same size and the same mean income, then accepting the Lorenz criterion is identical to assuming the transfers principle. For populations with different numbers of people but the same mean incomes, accepting the Lorenz criterion is identical to assuming the transfers principle and the symmetry axiom. For populations with differing numbers of people and differing mean incomes, acceptance of all three axioms is identical to acceptance of the Lorenz criterion. The scale invariance axiom allows us to express the *Y* axis in terms of percentages rather than total dollars. (Proofs of these results are not given here; see Atkinson, 1970; Dasgupta and others, 1973; Sen, 1973; Rothschild and Stiglitz, 1973; Kolm, 1976b.)

The Lorenz criterion provides a means of empirically testing whether, according to our three axioms, one distribution has more (or less) inequality than another. When two Lorenz curves do not cross, the Lorenz criterion is sufficient for determining which distribution has the greater equality.[6]

Incompleteness of the Basic Theory

Although the basic theory, through the transfers principle and its generalization to the Lorenz criterion, provides a method of comparing distributions, it is incomplete. Some discussions of inequality obscure the theory's incompleteness by failing to consider explicitly distributions whose Lorenz curves cross. This error of omission may reflect an unchecked assumption (built into some

[6] However, by its very nature the Lorenz criterion cannot indicate how much more or less equal one distribution is than another.

economic theories on the distribution of income—see our critique of the SDL below) that Lorenz curves rarely cross. When, however, the Lorenz curves for two distributions do cross, as in Figure 2, there is no way of using the Lorenz criterion to determine which distribution has more inequality.

Table 5 presents the number of times that Lorenz curves for different pairs of countries cross each other (Kuznets' data). Of the 66 possible pairs of curves, 50 pairs cross each other. In only 16 of the cases can we use the Lorenz criterion to determine which country has the more unequal distribution of income.[7] Our experience has been that the Kuznets data are in no way unusual in this respect. Thus, if we are to have a theory of wider application, either the basic theory must be extended or a more general theory must be developed.

[7] Atkinson (1970) shows that, given two distributions with crossing Lorenz curves, we can always find two measures of inequality satisfying our three axioms that rank the distributions differently.

Figure 2. Lorenz curves for the United States and India.

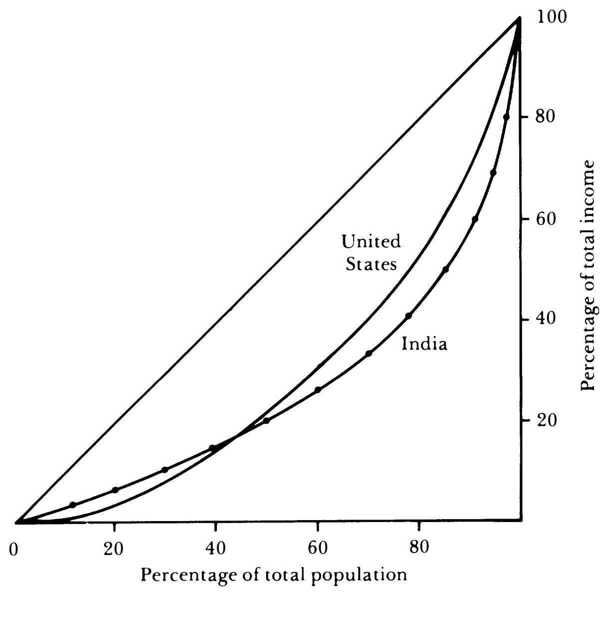


TABLE 5
Number of Times That Pairs of Lorenz Curves Cross

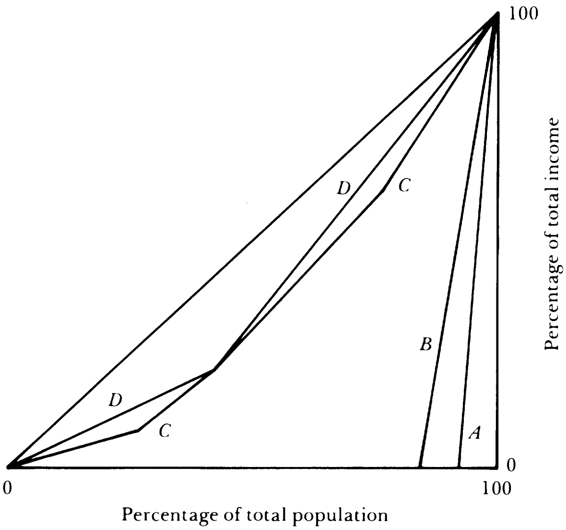
Country	United States	Italy	Puerto Rico	Denmark	Sweden	Netherlands	West Germany	India	Barbados	Ceylon	Mexico
Great Britain	1	1	1	1	1	0	0	1	0	0	0
United States	—	1	1	1	1	0	0	1	0	1	0
Italy	—	—	1	1	1	0	1	1	1	0	0
Puerto Rico	—	—	—	1	1	2	2	1	1	0	0
Denmark	—	—	—	—	2	1	1	1	1	1	1
Sweden	—	—	—	—	—	1	1	1	1	1	1
Netherlands	—	—	—	—	—	—	1	1	1	1	1
West Germany	—	—	—	—	—	—	—	1	3	1	1
India	—	—	—	—	—	—	—	—	1	1	0
Barbados	—	—	—	—	—	—	—	—	—	1	1
Ceylon	—	—	—	—	—	—	—	—	—	—	0
Mexico	—	—	—	—	—	—	—	—	—	—	—

CRITIQUE OF TRADITIONAL MEASURES

We have noted that the traditional measures of inequality do not rank distributions consistently with one another. Here we examine whether the traditional measures are consistent with the basic theory as outlined above. The coefficient of variation and the Gini coefficient are consistent with the basic theory, although they have other properties (discussed below) that many would consider undesirable. On the other hand, neither the standard deviation of the logarithm nor the mean relative deviation is consistent with the transfers principle.

The SDL, which is particularly sensitive to inequality at the lower end of the distribution, may fail to rank distributions correctly when they differ at the top of the distribution. Assume, for example, that we have ten people, nine of whom have \$1 apiece and one of whom has \$1 million (distribution *A* in Figure 3). We now transfer half of this last person's money to one of the other people (distribution *B*). Before the transfer SDL = 1.80; after the transfer SDL = 2.28, indicating that income inequality has increased.

Figure 3. How the SDL and MRD are inconsistent with the Lorenz criterion.



According to the transfers principle and the Lorenz criterion, however, inequality has decreased.

We surmise that continued use of the SDL is based on two premises. First, its statistical properties are well understood and the appropriate procedures for finding confidence intervals or testing for significant differences are known. The second premise is a belief among economists that the distribution of income is approximately lognormal (see Pen, 1971). If this were true, the SDL would be an entirely appropriate measure of dispersion (or inequality). Lorenz curves of lognormal distributions do not cross, and the SDL and the Lorenz criterion rank all lognormal distributions identically. The problem is that in the Kuznets data crossing Lorenz curves are the rule rather than the exception. Few income distributions in these data are lognormal. Schwartz (1978) presents additional evidence that the distribution of income is not lognormal. We conclude that the SDL is not an appropriate measure of inequality.

The MRD fails to satisfy the transfers principle for different reasons. Any redistribution among those above (or below) the mean is not reflected by a change in the mean relative deviation. The mean relative deviation is affected only by transfers from people above to people below the mean and vice versa. For example, imagine four people with incomes of \$20, \$40, \$60, and \$80 (distribution C in Figure 3). Assume that the second person transfers \$10 to the first and the fourth transfers \$10 to the third so that the incomes are now \$30, \$30, \$70, and \$70 (distribution D). Although the Lorenz curve of the second distribution is never below the Lorenz curve of the first distribution, the MRD (which depends only on the maximum distance of the Lorenz curve from the 45-degree line of perfect equality) is equal to 0.40 for both distributions. No change in inequality is indicated, although the two transfers of \$10 have certainly reduced inequality.

The insensitivity of the MRD to some transfers is reflected in yet another defect. The effect on the MRD of a transfer of some fixed amount from one person above the mean to a second person below the mean does not depend on how far either person is from the mean. In the foregoing example, a transfer of \$1 from the person with \$60 to the person with \$40 would reduce the MRD by exactly the same amount as a \$1 transfer from the person with \$80 to the person with \$20, although the latter transfer intuitively reduces inequality by more than the former.

Principle of Diminishing Transfers

This last criticism of the MRD implies that a measure of inequality should take into account the differential impact of transfers between different points in the distribution. Suppose we compare a transfer of \$5,000 from a person who has \$125,000 to a person who has \$100,000 with a transfer of \$5,000 from someone with \$25,000 to someone who has no money. Atkinson (1970), Kolm (1976b), and others have argued that the second transfer reduces inequality more than the first. Formally, consider two persons with incomes of X and Y, with X less than Y. The principle of diminishing transfers (Kolm's terminology) states that the reduction in inequality attributable to a transfer from the person with income X to another person with income $X - C$ (where $0 < C < X$) is greater than the reduction attributable to an equal transfer from the person with income Y to someone with income $Y - C$.

The justification for this axiom is based on the view that as people's absolute incomes increase, the difference between them becomes less important because their *relative* shares become more equal. Whether this principle is appropriate to the conception of inequality in the distribution of variables other than income is debatable.

Effect of Transfers on the CV and GC

The coefficient of variation has been criticized for giving equal weight to transfers at all levels.[8] The first derivative of CV with respect to a transfer t to a person with income Y_i from a person with income Y_j is

$$d(\mathrm{CV})/dt = [1/\mathrm{CV}(n\bar{Y})](Y_i - Y_j)$$

The effect of the transfer is thus proportional to the difference in income between the person giving the money and the person receiving it. As long as the transfer is from a richer person (j) to a poorer person (i), the coefficient of variation will decrease, thus satisfying the transfers principle. However, it violates the principle of diminishing transfers because the decrease depends only on the difference between the two incomes and not on the absolute amounts of income (or their ratio to the mean). While this may make the CV appropriate for measuring inequality in other variables, it reduces its usefulness as a measure of income inequality.

It is clear that the GC ranks distributions whose Lorenz curves do not cross in the same order as the Lorenz criterion. This is easily seen from the graphic interpretation of the GC described in the appendix. But while the GC is reduced by any transfer from a richer person to a poorer person, the size of the reduction is a linear function of the number of people with incomes between these two.[9]

[8] The meaning of "equal weight to transfers at all levels" is slightly ambiguous. This ambiguity is resolved in our later distinction between absolute transfer neutrality (to which the present discussion refers) and relative transfer neutrality.

[9] This follows from noting that the Gini coefficient can be expressed as:

$$(1/\bar{Y}n^2) \sum_{i=1}^{n} (2i - n - 1)Y_i$$

where i is the rank order of the individual in the income distribution (Rothschild and Stiglitz, 1973).

Atkinson (1970) notes that if the distribution of income is unimodal, then transfers among people in the middle of the distribution will be given more weight than transfers at either end. Thus, like the CV, the GC does not satisfy the principle of diminishing transfers. This problem is also illustrated by the fact that as a richer person transfers money to a poorer person, the effect of each additional dollar on the GC diminishes only if the richer person or the poorer person changes rank in the distribution. Further criticisms of the GC are given in Rothschild and Stiglitz (1973) and Theil (1967).

The principle of diminishing transfers states that the effect of a transfer on a measure of inequality should depend on the incomes of the people giving and receiving the transfer: The greater the difference in their incomes and the lower down in the distribution they are, the greater the effect of the transfer should be. However, it does not state *how much* greater the effects of such transfers should be. While many analysts would accept the principle of diminishing transfers, we doubt that there is a consensus on this latter issue of magnitude. Furthermore, as we shall demonstrate below, by making different choices one can create measures of inequality that are as inconsistent with each other as are the traditional measures.

As noted above, our objection to the CV is that the effect of a transfer depends only on the difference in incomes between the receiver and giver ($Y_i - Y_j$) and does not increase when Y_i and Y_j are nearer the bottom of the distribution. If there were a family of measures of inequality (defined by a parameter p) in which the effect of a transfer on a measure of inequality were proportional to [(signum p)($Y_i^p - Y_j^p$)], then these measures would be:

1. Consistent with the principle of diminishing transfers whenever $-\infty < p < 1$.
2. Equal to the coefficient of variation when $p = 1$.
3. Consistent with a principle of *increasing* transfers whenever $p > 1$.

Atkinson has proposed such a family of measures. While the selection of different values of p can be interpreted solely in terms of a decision as to how much more weight should be given to transfers at the bottom of the distribution, the discovery of this family of measures evolved from the welfare approach to inequality, which we now discuss briefly.

THE WELFARE APPROACH

Economists have attempted to develop a general theory of inequality that is consistent with the basic theory presented earlier and that would allow us to deal with all situations where Lorenz curves cross. Their approach has been to base the measurement of inequality on a theory of social and individual welfare.

Dalton (1920) was perhaps the first to argue that economists were interested not in inequality per se but in the effects of inequality on economic welfare. As he put it: "The objection to great inequality of incomes is the resulting loss of potential economic welfare." This argument has been used to justify the development of a general theory based on notions of individual and societal welfare. Dalton goes on to suggest that the degree of inequality in a distribution should be measured by the loss in welfare that it causes.

By individual welfare an economist means one's sense of well-being, one's happiness or satisfaction with life, or one's potential (given one's resources) for obtaining these things.[10] In the literature on income inequality a standard theoretical assumption is that the relationship between income and well-being (the welfare function) is the same for everyone. Economists also assume that increasing a person's income increases his or her welfare and that the effect of income on an individual is independent of other resources that person might possess. This is equivalent to assuming that the individual well-being function is of the form $g(X) + f(Y)$, where Y represents the income possessed by the individual[11] and X represents that person's other resources. Finally, it is assumed that

[10] Today economists no longer equate the concepts of individual welfare and utility. For an excellent discussion of the history of the tension between these two concepts in economics, see Schumpter (1954).

[11] Rothschild and Stiglitz (1973) suggest difficulties that can arise from using only income as a measure of welfare when different countries have different price structures and public services. Intertemporal changes in inequality within a society can also be affected by changes in relative prices.

the level of well-being an individual possesses is determined by the amount of his or her income, independent of the amount of income possessed by others.[12]

Besides using a notion of individual welfare, economists also use a notion of social welfare. Social welfare is measured by a function S, which represents society's notion of how fair or desirable a particular distribution is. This measure S may be a function of individual welfares ($g(X) + f(Y)$), the part of individual welfare due to income alone ($f(Y)$), or Y (the incomes that individuals receive). In the first case S is a measure of the desirability of the distribution of complete individual welfare; in the second, that of the welfare due to income; and in the third, that of income. Only the last two formulations have been extensively considered in the literature. It is assumed that S increases as income increases. That is, if we increase everyone's income, social welfare is increased, implying that for cases where income is distributed equally, S ranks distributions in order of their mean incomes.

A specific form of S is of particular interest to economists: the additive social welfare function $S = \Sigma_{i \in P} f(Y_i)$, in which social welfare is the sum of individual welfares. This form of S assumes that the welfare gained by society from each individual's welfare is independent of the welfare of other individuals. This is a generalization (to the societal level) of the individual independence assumption. The desirability of a particular distribution when measured by an additive S has nothing to do with justice. Desirability is defined only in terms of maximizing the sum of individual welfare.[13] This social welfare function is consistent with the intellectual tradition of the utilitarians. While there is nothing egalitarian about an additive social welfare function (see Sen, 1973), the assumption of declining marginal individual welfare functions is sufficient to

[12] Sociological criticism of this absolutist theory of the relationship between welfare (well-being) and income has a long history that goes back to Marx, de Tocqueville, and Durkheim. Probably the most developed criticism is the theory of relative deprivation (Merton, 1968; Stouffer and others, 1949). For a recent discussion of the issue see Easterlin (1974) and Duncan (1975).

[13] For a discussion of the independence assumption and its substantive importance for the distribution of income see Harsanyi (1955), Strotz (1958, 1961), and Fisher and Rothenberg (1961, 1962).

assure that, for a given total income, egalitarian distributions will result in greater social welfare than inegalitarian distributions.

There are severe problems in constructing a general social welfare function from individual welfare functions, especially if individual welfare functions can reflect attitudes about how income ought to be distributed, as well as about the well-being received from the particular income possessed by an individual. (See Arrow, 1963, for a discussion of his "impossibility theorem.") Hamada (1973) has shown that these same problems exist in the specific case of income inequality. Nonetheless, we surmise that sociologists will find the additive social welfare function a useful model—partly because it is the simplest functional form consistent with the standards to be met by a good measure of inequality.

MEASURES OF INEQUALITY

We now define measures of inequality based on functions for individual and social welfare. Dalton (1920) suggested measuring inequality as the loss in welfare that results from inequality. Let $S(\mathbf{Y})$ be the amount of welfare that exists when income is distributed as the vector $\mathbf{Y}$ and let $S^*(\mathbf{Y})$ be the amount of welfare that exists when income $\mathbf{Y}$ is distributed equally. We make the assumption that income can be redistributed without cost or loss in total income. Dalton's measure of inequality is $I^* = 1 - S/S^*$. We can interpret this measure as the percentage of total potential welfare (obtainable when income is distributed equally) that is lost due to income inequality. If S is an additive function of individual welfares, then I^* becomes $I_a^* = 1 - \Sigma f(Y_i)/nf(\overline{Y})$. If, as Dalton hypothesized, the individual welfare function is of the form $f(Y) = a + bY^p$ (with p less than 1, for reasons that will soon become clear), then I_a^* can be written:

$$I_a^* = 1 - \Sigma[a + bY_i^p/n(a + b\overline{Y}^p)] \tag{1}$$

This measure will be zero when income is equally distributed and positive otherwise. Its upper limit is 1.

Atkinson (1970) points out that Dalton's inequality measure, I^*, makes very strong assumptions about the measurability of social and individual welfare. We must be able to measure both social and individual welfare with a ratio scale.[14]

Atkinson provides us with a way to make weaker assumptions about the measurability of welfare. He suggests measuring the ratio in Dalton's formula in income units (which is a ratio-level variable) rather than in welfare units.

It was noted earlier that distributions where income is uniformly distributed are ranked in the same order by their mean as by S. If S is continuous in income, then we can use these mean incomes as an indicator of the level of welfare. This is the idea behind Atkinson's notion of equally distributed income equivalents. For any given distribution (vector) of income $\mathbf{Y}$, we identify the particular uniform distribution of income $\mathbf{Y}'$, which has the same total social welfare as $\mathbf{Y}$: $S(\mathbf{Y}) = S(\mathbf{Y}')$. Since all incomes in $\mathbf{Y}'$ are equal to each other, Atkinson calls their mean, Y', the equally distributed income equivalent of $\mathbf{Y}$. Since Y' is the mean of a uniform distribution, it can be used to rank distributions in the same order as S. When there is no inequality in $\mathbf{Y}$, $\mathbf{Y}'$ will equal $\mathbf{Y}$ and Y' will equal the mean of the distribution, $\bar{Y}$. Dalton's measure, I^*, can now be redefined as $I' = 1 - Y'/\bar{Y}$. The numerator is the amount of welfare measured in income units associated with the given distribution, $\mathbf{Y}$. The denominator is the amount of potential welfare that would result from distributing this income equally, again measured in income units. Our new measure may be interpreted as that percentage by which we could reduce current total (or average) income and still maintain the same level of welfare if income were equally distributed in the process. Our measure will be equal to zero if income is equally distributed; it will approach 1 the more unequally income is distributed. One consequence of the population

[14] Dalton (1920) and Atkinson (1970) neglect to note that if a does not equal zero in the individual welfare function, $f(Y) = a + bY^p$, then I_a^* will violate the scale invariance axiom. If a does equal zero, then I_a^* becomes a one-parameter family of inequality measures and is equivalent, at the ordinal level, to Atkinson's measures (defined later on). Dalton and Atkinson also fail to note that if $f(Y) = a + b \log Y$ (another welfare function proposed by Dalton), then I_a^* will violate the scale invariance axiom (even when a equals zero).

symmetry axiom is that the maximum possible value of our measure depends to a limited extent on the population size.

The advantage of using equally distributed income equivalents is that it allows us to make weaker assumptions about the measurability of welfare. Social welfare only needs to be ordinally measurable; if we can rank societies according to their level of social welfare, then we shall be able to rank them in terms of inequality. If social welfare is additive, then I' becomes:

$$\begin{aligned} I'_a &= 1 - f^{-1}[\Sigma f(Y_i)/n]/f^{-1}(f(\bar{Y})) \\ &= 1 - f^{-1}[\Sigma f(Y_i)/n]/\bar{Y} \end{aligned}$$

In this case we need only be able to measure individual welfare in terms of an interval scale; we need only know the relationship between income and welfare to within a linear transformation. Therefore I'_a will usually depend on fewer parameters than I^*_a.

CONSISTENCY WITH THE BASIC THEORY

In the last section we presented four measures: Dalton's measure, $I^* = 1 - S/S^*$; Atkinson's redefinition of Dalton's measure in terms of equally distributed income equivalents, $I' = 1 - Y'/\bar{Y}$; the special case of I^* where S is an additive function of individual welfare, $I^*_a = 1 - \Sigma f(Y_i)/nf(\bar{Y})$; and the special case of I' where S is an additive function of individual welfare, $I'_a = 1 - f^{-1}[\Sigma f(Y_i)/n]/\bar{Y}$. What properties must S, Y', and $f(Y)$ possess in order for these four measures to be consistent with the basic theory? We shall examine properties necessary and sufficient for satisfying the transfers principle, the population symmetry axiom, and the scale invariance axiom.

Consistency with the Transfers Principle

In order for I^* and I' to satisfy the transfers principle, it is necessary and sufficient that S and Y' satisfy a very weak concavity property. The mathematical term is strict Schur concavity (see Kolm, 1976a); Rothschild and Stiglitz (1973) have termed this property "locally equality preferring." If S satisfies this property, then any monotonic function of S, including Y', will also satisfy it.

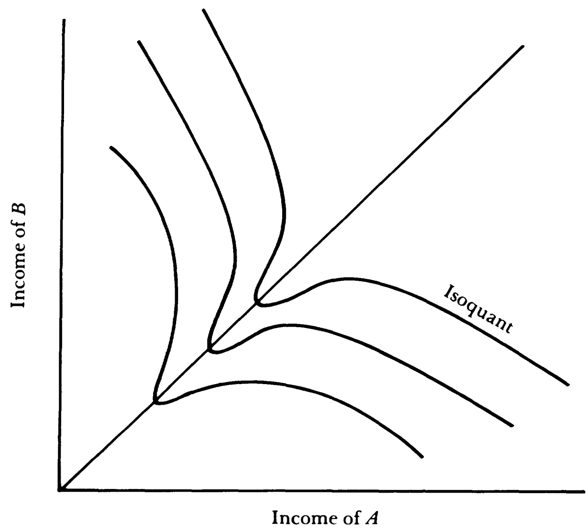
The definition (Rothschild and Stiglitz, 1973) is: A function $S(\mathbf{Y})$ is strictly "locally equality preferring" if, for every vector $\mathbf{Y}$ and pair of subscripts j, k such that $Y_j \neq Y_k$,

$$S(\mathbf{Y}) < S[a\mathbf{Z} + (1 - a)\mathbf{Y}] \qquad \text{for } 0 < a < 1$$

where $Z_i = Y_i$ for $i \neq j$, k and $Z_j = Z_k = (Y_j + Y_k)/2$.

Thus $\mathbf{Z}$ is a modified vector $\mathbf{Y}$ in which j and k have equalized their incomes; that is, there has been a transfer of income from the richer to the poorer. The terms in brackets on the right side of the inequality represent the case where there is *some* transfer between j and k (from the richer to the poorer) and where everyone else's income has remained the same. Schur concavity means that isoquants (Figure 4) representing levels of social welfare must be increasing from the origin and that any line perpendicular to the 45-degree line can cross an isoquant only twice—once on either side of the 45-degree line (Rothschild and Stiglitz, 1973). The more

Figure 4. Schur concavity (from Rothschild and Stiglitz, 1973).



traditional notions of strict quasi-concavity and strict concavity[15] are sufficient but not necessary conditions for I^* and I' to satisfy the transfers principle. (Proofs are found in Rothschild and Stiglitz, 1973, for I^* and in Kolm, 1976b, for I'.)

If S is an additive social welfare function, then the second derivative of individual welfare, $f(Y)$, must be negative in order for I_a^* and I_a' to satisfy the transfers principle. (Previously we assumed that the first derivative is positive.) This restriction on the second derivative of $f(Y)$ is mathematically equivalent to the previously discussed assumption of the diminishing marginal utility of income. The sum of all individual welfares is increased by reducing inequality because in taking a dollar away from one person and giving it to a poorer person, we decrease the first person's welfare by less than we increase the poorer person's welfare. Note that this single restriction of $f(Y)$, required for I_a^* and I_a' to satisfy the transfers principle, also assures that they will satisfy the principle of diminishing transfers.

Consistency with the Population Symmetry Axiom

A sufficient condition for I^* and a necessary and sufficient condition for I' to satisfy the symmetry axiom is the following: If we have r populations of the same size such that each population has the same distribution of income ($\mathbf{Y}$), then S must have the property $S(\underline{\mathbf{Y}}) = rS(\mathbf{Y})$, where $\underline{\mathbf{Y}}$ is the total distribution of income across the r populations. The proof is straightforward. Since additive S necessarily satisfies this property, I_a^* and I_a' will always satisfy the symmetry axiom for populations.

Consistency with the Scale Invariance Axiom

A sufficient condition for I^* to be consistent with the scale invariance axiom is that S be homogeneous of any degree. A function S is homogeneous of degree P if $S(\lambda\underline{\mathbf{Y}}) = \lambda^P S(\mathbf{Y})$ for all $\mathbf{Y}$ and λ. If we increase everyone's income by a factor λ, social welfare will increase by λ^P. Then S^* will also increase by λ^P and therefore

[15] A function $f(X)$ is strictly concave if for any two distributions X, Y such that $X \neq Y$, $f(aX + (1 - a)Y) > af(X) + (1 - a)f(Y)$ for $0 < a < 1$. A function $f(X)$ is strictly quasi-concave if for any two distributions X, Y such that $f(X) > f(Y)$, $f(X) > f(Z)$ for all $Z = aX + (1 - a)Y$; $0 < a < 1$.

$I^*(\lambda\mathbf{Y}) = I^*(\mathbf{Y})$. If $P > 1$, welfare will increase faster than income. If $P < 1$, welfare will increase more slowly.

If I' is to satisfy the scale invariance axiom, then Y' must be linear homogeneous (that is, of degree 1). This follows directly from the fact that $\overline{Y}$ is linear homogeneous. In order for Y' to be linear homogeneous, S must be homothetic. This is a weaker condition than requiring S to be homogeneous (as is required for I^*). Hence S is homothetic if $S(\lambda\mathbf{X}) = S(\lambda\mathbf{Y})$ whenever $S(\mathbf{X}) = S(\mathbf{Y})$. (See Kolm, 1976b, for further discussion.)

If I_a^* is to satisfy the scale invariance axiom, f must have a very particular form: $f(Y) = bY^{1-e}/(1 - e)$. For I'_a to satisfy this axiom, f can take a slightly more general form: $f(Y) = a + bY^{1-e}/(1 - e)$; or, when $e = 1, f(Y) = a + b \log Y$.[16] Note that in each of these equations b must be positive in order for $f(Y)$ to be an increasing function of Y, and e (not to be confused with the base of the natural logarithms) must be positive in order for I_a^* and I'_a to satisfy the transfers principle. (Proof of this is given in Kolm, 1976*a*.) Note also that I_a^* is independent of b and I'_a is independent of both a and b. Thus both I_a^* and I'_a depend on only one parameter (e) and are, in fact, increasing monotonic transformations of each other (except when $e = 1$, in which case I_a^* is undefined). Although they are therefore equivalent at the ordinal level, they are known as Dalton's measure and Atkinson's measure respectively (see Atkinson, 1970).

We have arrived at a very strong result: I_a^* and I'_a will be consistent with the basic theory if and only if the individual welfare function is a power function. Furthermore, only the choice of the power will affect the measures of inequality. The only assumption we have made which goes beyond the basic theory is that welfare is additive. Because of their special properties and their increasing use in empirical analysis, we now discuss these measures in detail.

Atkinson's Measure

In the last section we pointed out that Dalton's and Atkinson's measures are the only ones based on an additive social

[16] As e approaches 1, the limiting behavior of $Y^{1-e}/(1 - e)$ is log Y. As stated in footnote 14, this special form of $f(Y)$ cannot be used with I_a^*.

welfare function that are consistent with our basic theory. Dalton's measure was earlier represented in terms of a ratio of actual total individual welfare to potential total individual welfare:

$$I_a^* = 1 - \Sigma Y_i^{1-e}/n\overline{Y}^{1-e} \qquad \text{for } e > 0 \text{ and } e \neq 1 \tag{2}$$

Atkinson's measure is expressed in terms of equally distributed income equivalents:

$$I_a' = 1 - [(\Sigma Y_i^{1-e}n)^{1/(1/e)}/\overline{Y}] \qquad \text{for } e > 0 \text{ and } e \neq 1 \tag{3}$$

$$I_a' = 1 - \{\exp[\Sigma(\log Y_i)/n]/\overline{Y}\} \qquad \text{for } e = 1 \tag{4}$$

Equations (2) and (3) give the same rankings, since one is a strictly increasing monotonic function of the other.

The core of Atkinson's measure is the ratio between a generalized mean and the standard arithmetic mean for a distribution. Thus in Formula (4) Atkinson's measure is the ratio between the geometric mean and the arithmetic mean. For $e > 0$ the generalized mean is smaller than the arithmetic mean (assuring that inequality will be positive) except where income is distributed equally, in which case they will be equal and I_a' will equal zero.

Extensions of Atkinson's Measure

To date, Atkinson's measure has been defined only for positive values of e. In this section we offer an interpretation of inequality measures based on values of $e \leq 0$.

Atkinson (1970) notes that if $e = 0$, Formula (3) will always equal zero, regardless of how unequal the income distribution is, and that the proportional loss in social welfare attributable to inequality is therefore zero. While this is true, it is informative to examine the behavior of Formula (3) when e is close to zero. By calculus it can be shown that for values of e near zero, Formula (3) is approximately equal to:

$$(e)(1/n)\Sigma(Y_i/\overline{Y} \log Y_i/\overline{Y}) \tag{5}$$

Apart from the scaling factor e, this is equivalent to Theil's (1967) measure of inequality.

What about negative values of e? They have no place in the "welfare approach" because they violate the principle of transfers and the principle of diminishing transfers. If we completely reverse our perspective, however, the theory may be reformulated in terms of individual and social "decadence"[17] instead of welfare. We can summarize the situation in which the social decadence function S is additive and $f(Y)$ is the individual decadence function as follows. To begin with, the first derivative of f should be positive, indicating that decadence increases as income increases. The second derivative of f will also be positive, indicating that the marginal decadence of income increases with income. This assures that the principle of transfers will be satisfied and that social decadence will be minimized by an equal distribution of income. The measure of inequality is defined as

$$I_a^* = \Sigma f(Y_i)/nf(\overline{Y}) - 1 \qquad \text{for } e < 0$$

and is interpreted as the proportionate increase in social decadence attributable to inequality. Finally, in order to satisfy the scale invariance axiom and the first- and second-order conditions, $f(Y)$ must have the form $f(Y) = bY^{1-e}$, where b is positive and e is negative. If inequality is measured in terms of equally distributed income equivalents, then $f(Y)$ can have the form $f(Y) = a + bY^{1-e}$. The formula for inequality (analogous to Formula (3) above) then becomes:

$$I_a' = (\Sigma Y_i^{1-e}/n\overline{Y}^{1-e})^{1/(1-e)} - 1 \qquad \text{for } e < 0 \tag{6}$$

Note that when e equals -1, Formula (6) is equivalent to the coefficient of variation.

The assumptions and the potential weaknesses of this "decadence approach" to measuring inequality are analogous to those of the welfare approach. We are now in a position to make a very strong claim, however. Any measure of inequality that is an increasing monotonic function of a sum of functions of individual incomes and satisfies the three axioms of the basic theory *must* be equivalent at the ordinal level to one of the measures in Formulas

[17] This alternative approach is consistent with a puritan or otherworldly perspective in which income is seen as having undesirable consequences for both the individual and society. Some might wish to substitute concepts of individual and social anomie (or tension) for decadence. Our purpose is to demonstrate the logical consistency of this alternative approach with the basic theory of inequality.

(3) through (6).[18] (The proof of this is analogous to that given in Kolm, 1976a.) Therefore, as long as the basic theory is accepted, or until general procedures are developed for incorporating the dependence of one individual's welfare (or decadence) on another's welfare, social scientists should probably select their measures of inequality from the family of Atkinson's measures or monotonic transformations thereof.

Interpretations of Atkinson's Measure

Welfare Interpretations. How are we to think of e? From the perspective of the welfare approach, e is the parameter of the individual welfare (decadence) function that determines the rate at which welfare (decadence) increases in response to a change in income. (Note that although $f(Y)$ can also depend on parameters "a" and "b," Aktinson's measure of inequality depends only on the parameter e.)

A second interpretation of e, still based on the welfare approach, emphasizes the social welfare effect of inequality. Larger values of e imply a greater aversion to having people who are poor relative to the mean.[19] As $|e|$ increases, the value of Atkinson's measure will also increase, indicating a larger decrease (increase) in

[18] While some sociologists might find the additive social welfare function unacceptable, most measures of income inequality are in fact monotonic transformations of a sum of functions of individual incomes. Thus the MRD can be written as $(1/n)\Sigma\,|(Y_i/\overline{Y}) - 1|$; the CV as $[(1/n)\Sigma(Y_i/\overline{Y})^2 - 1]^{0.5}$; and the SDL as $[(1/n)\Sigma(\log Y_i)^2 - Z^2]^{0.5}$, where $Z = \Sigma \log Y_i/n$. Only the Gini coefficient, which is a function of the differences between all pairs of incomes, deviates from the assumption that inequality is a sum of functions of individual incomes.

[19] Atkinson (1970) suggests concepts of absolute and relative inequality aversion and notes their parallel to analogous concepts of risk aversion in economic theories of uncertainty. Relative inequality aversion is measured by the product of income and the ratio of the second derivative to the first derivative of the individual welfare (decadence) function. For all the individual welfare functions that are consistent with the additive social welfare function and the basic theory, this ratio equals negative e. Thus, for any given individual welfare function, relative inequality aversion is constant across all levels of income. However, this constant varies between functions and can be measured by e. As e increases, individuals become more willing to give money to those who are poorer (possibly imagining that they themselves might someday be among the poor) and less willing to help richer persons become even richer, even if the financial gain of the beneficiary exceeds the donor's cost.

social welfare (decadence) due to inequality in the distribution. Thus for the Kuznets data on the United States, for values of e of 0.5, 1.0, 2.0, and 4.0, respectively, Atkinson's measure (the equally distributed income equivalent version) takes on the values of 0.1296, 0.2425, 0.4204, and 0.6050.

A third interpretation of e, also suggested by Atkinson (1975), is perhaps more realistic and does not require the complete welfare theory apparatus. In the earlier discussion of the welfare approach, we unrealistically assumed that income could be transferred without any cost or loss in total income. If we admit the existence of administrative and possibly other costs, we can ask ourselves how "efficient" a transfer must be in order for it to be worthwhile. Atkinson suggests that we imagine two persons, one of whom has twice the income of the other. If the richer person donates \$1, what percentage (t) of this dollar must the poorer person receive in order for the transfer to be worthwhile (that is, in order that the poorer person's gain will match the richer person's loss)? Clearly, society will only want to transfer money if the net gain in welfare is positive. Whatever the value of t, the corresponding value of e is $e = -\ln(t)/\ln 2$. Alternatively, if we want to interpret a particular value of e, we can find its implied transfer efficiency by the formula $t = 2^{-e}$. Thus if e equals 1, it is only necessary that 50 percent of the transfer be received. Other positive values of e can be similarly interpreted.

Nonwelfare Interpretations. The fourth interpretation of e is perhaps the simplest and least theoretically constrained. Since for all values of e these measures of inequality are consistent with the Lorenz criterion and the basic theory, the ranking of distributions whose Lorenz curves do not cross will be the same for all values of e. Therefore one's choice of e only influences comparisons among distributions whose Lorenz curves cross. As e increases, more and more weight is put on the share of income possessed by the bottom portion of the population (represented by the lower portion of the Lorenz curve). Similarly, lower values of e weight the upper portion of the Lorenz curve more heavily. As an illustration, we compare the United States and India (see Figure 2). Since the bottom segment of the population has a larger share of the total income in India than in the United States, we find that for sufficiently large values of e, India has a more equal distribution of income. For India, Atkinson's measure has the values of 0.1878, 0.2950, 0.3973, and 0.4751 when e equals 0.5, 1.0, 2.0, and 4.0, respectively. Atkinson's measure has the same value for India and the United States when e is approximately 1.75. For lower values of e the United States is considered to have a more equal distribution of income; for values greater than 1.75 India is considered to have a more equal distribution of income. As e increases, incomes at the bottom are weighted more heavily, as may be seen by letting e go to infinity. In this case pairs of distributions will be ranked by the lowest point at which the associated Lorenz curves diverge, which generally corresponds to the difference between the incomes (divided by their respective means) of the poorest person in each distribution (Hammond, 1975). Similarly, if e goes to negative infinity, distributions will be ranked by the highest point at which the Lorenz curves diverge. If two Lorenz curves cross exactly once, there will be a cutoff value of e such that the Lorenz curve which is higher at the bottom will be judged less unequal whenever e is greater than the cutoff and more unequal whenever e is less than the cutoff.

Our final interpretation of e is a reformulation of the previous one, but from a slightly different perspective. Deciding how heavily to weight different portions of the Lorenz curve is formally equivalent to deciding where in the distribution transfers are most important. Prior to the discussion of the welfare approach, we introduced a fourth axiom—the principle of diminishing transfers—stating that the effect of a transfer between two people whose incomes differ by a specified amount should be inversely related to their absolute position in the population. We suggested the existence of a family of measures of inequality that would allow the user to make a normative judgment as to how strong the principle of diminishing transfers should be. The measures specified by Formulas (3) through (6) do exactly this. The marginal effect of a transfer to a person with income Y_i from another with income Y_j is proportional to

$$\begin{array}{ll} Y_j^{-e} - Y_i^{-e} & \text{for } e > 0 \\ \log(Y_i/Y_j) & \text{for } e = 0 \\ Y_i^{-e} - Y_j^{-e} & \text{for } e < 0 \end{array}$$

We observed earlier that when $e = -1$, Atkinson's measure of inequality equals the coefficient of variation—the effect of a transfer depends only on the absolute difference between Y_i and Y_j, not on their position in the distribution. We refer to this latter property as *absolute transfer neutrality*. The CV has the highest (least negative) value of e that fails to satisfy the principle of diminishing transfers. When e is less than -1, inequality will be consistent with the principle of increasing transfers, which emphasizes the importance of transfers *from* the rich (rather than *to* the poor), presumably in order to reduce individual and social "decadence." When e is greater than -1, the principle of diminishing transfers is satisfied, with increasing emphasis placed on transfers to the poor as e increases. When e equals zero, we use Theil's measure and find that the effect of a transfer on inequality depends only on the ratio of the receiver's income to the giver's income. This property is referred to as *relative transfer neutrality*. For values of e greater than zero (that is, those which are consistent with the welfare theory approach), measures of inequality are also consistent with the principle of *relative* diminishing transfers. This principle states that the effect of a transfer to one person from someone whose income is a fixed proportion higher (say double) diminishes as the absolute level of their incomes increases. For example, a small transfer to a person with $7,500 from someone with $15,000 is more effective in reducing inequality (or increasing social welfare) than a transfer of the same size to a person with $15,000 from someone with $30,000. (Obviously, relative transfer neutrality implies that these two transfers would have the same impact on inequality.) Finally, when e is between -1 and 0, the principle of diminishing absolute transfers is satisfied but not the principle of diminishing relative transfers.

We have discussed several interpretations of e. Sociologists may be reluctant to place much stock in the first two interpretations because they are so deeply embedded in economic welfare theory. But, as the remaining interpretations demonstrate, Atkinson's measure and its parameter e can be interpreted from a more general and practical framework that does not require the researcher to accept the many assumptions of the welfare approach. This is not to say that one's choice of e can be value-free, since it depends on judgments about how different portions of the Lorenz curve should be weighted in a measure of inequality or how different types of transfers should affect a measure of inequality.

Selecting Values of e

How then is one to choose e from the range between positive and negative infinity? (Remember: If no Lorenz curves cross, then the choice of e is irrelevant.) Before addressing this question, we think it is important to reflect upon the intended use of the measure of inequality. Sen (1973) makes a useful distinction between descriptive and normative measures of inequality. Our viewpoint differs from Sen's, however. We do not think that the distinction lies between different measures, but rather between different uses of the same measure. Research questions such as "Has inequality declined in the United States over the past thirty years?" or "Is income distributed more equally in developed societies than in underdeveloped societies?" implicitly raise issues of fairness and justice. If the data are of sufficient quality and the Lorenz curves do not cross, then the answer is unambiguous. If, as we suspect will often be the case, Lorenz curves do cross, then a normative question can only be answered within the context of a normative definition of inequality. Earlier we raised the issue of whether inequality is a unidimensional or multidimensional concept. It seems to us that normative definitions of inequality are usually unidimensional. If this is the case, the researcher must choose that value of e, presumably based on at least one of the interpretations offered above, which best corresponds to his or her definition of inequality. In the event of uncertainty, it would be prudent to apply a range of values of e. In this way one can judge the sensitivity of results to systematic changes in the normative definition of inequality. If the principle of diminishing absolute transfers is assumed, then e must be greater than -1. If, in addition, the principle of diminishing relative transfers is assumed, then e must be greater than zero. We suspect that after reflecting on the different interpretations of e, most sociologists would agree that when using Atkinson's measure to address normative questions, e should be between -0.5 and 2.5.

Some attempts based on the standard welfare approach have been made to estimate empirically the appropriate value of e. Stevens (1959), Schwartz (1974), and Winship (1976) all estimate an equation relating individual welfare to income in terms of the functional form $W = a + bY^{1-e}$. Their estimates, using attitudinal survey data about the level of well-being associated with different levels of income, suggest that e should be between 0.5 and 0.75. Stern (1977) examines assorted data on individual consumer maximizing behavior and arrives at estimates ranging from 0 to 10 with a concentration around 2.

Research on inequality is not necessarily normative. We surmise that Kuznets' (1963) comparison of inequality between developed and underdeveloped countries was intended to be primarily descriptive. This would have been more obvious had he expressed his research question like this: "Which, if any, aspects of income inequality (for example, inequality at the bottom and top of the distribution) are associated with level of economic development?" It seems to us that this type of question is based on a multidimensional conception of inequality. It asks whether there are any consistent differences between the Lorenz curves of developed and underdeveloped countries; this is a multidimensional problem because a Lorenz curve cannot be adequately described by a single summary measure. To address this type of question, we would recommend analyzing the data using a wide range of values of e, including values that emphasize inequality at the top of the distribution. Such an analysis would indicate whether level of development is associated with inequality across the entire distribution, only in particular portions of it, or not at all. While from a normative perspective we would reject variants of Atkinson's measure based on values of e that are less than -1, it seems clear that they can be useful in addressing some descriptive research questions.

The economics literature has postulated that all measures of inequality incorporate, either explicitly or implicitly, some notion of social welfare. Earlier we examined the sensitivity of the traditional inequality measures to transfers among different segments of the population. It is informative to compare them with Atkinson's measure using different values of e. For the Kuznets data, the rank order given by the standard deviation of the logarithm of income is identical to the ordering produced by Atkinson's measure when e is between 1.81 and 1.84. The mean relative deviation and Gini coefficient correspond well to Atkinson's measure for values of e

between 0.55 and 0.95. Never are more than two pairs of countries ranked differently. The coefficient of variation, of course, corresponds exactly to $e = -1$.

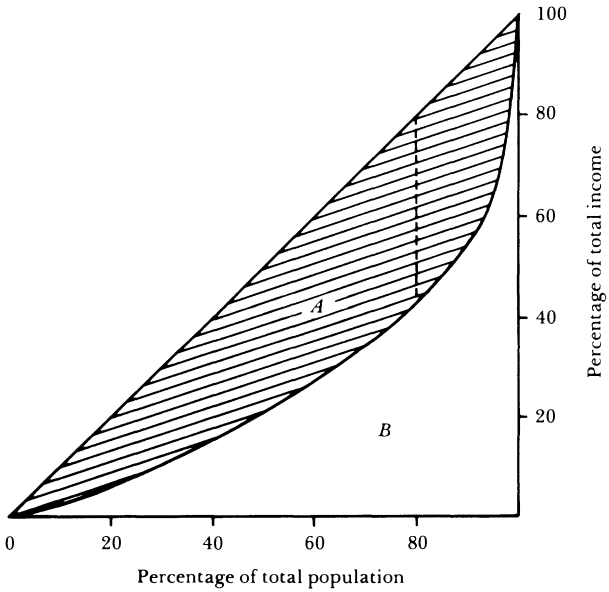
APPENDIX: FORMULAS FOR TRADITIONAL MEASURES OF INEQUALITY

The Gini coefficient is defined as one half of the average of the absolute differences between all pairs of relative incomes $(Y_i/\overline{Y})$:

$$GC = (1/2n^2) \sum_n \sum_n |Y_i/\overline{Y} - Y_j/\overline{Y}|$$

The Gini coefficient is directly interpretable in terms of the Lorenz curve. It is the ratio of the area between the Lorenz curve and the diagonal of equality to the total area under the diagonal. In Figure A-1, the Gini coefficient is equal to the area of segment A divided by the sum of the areas of segments A and B.

Figure A-1. Relationship of GC and MRD to the Lorenz curve.



The coefficient of variation is simply the standard deviation of income divided by its mean:

$$\mathrm{CV} = \sqrt{\sum_n (Y_i - \bar{Y})^2/n}/\bar{Y}$$

In terms of the Lorenz curve, the coefficient of variation is equal to the standard deviation of the slope of the curve.

The standard deviation of the logarithm of income is given as

$$\mathrm{SDL} = \sqrt{\sum_n (\log Y_i - Z)^2/n}$$

where Z is equal to $\Sigma_n (\log Y_i)/n$.

The mean relative deviation is given by the formula

$$\mathrm{MRD} = (1/2n) \sum_n |Y_i - \bar{Y}|/\bar{Y}$$

The mean relative deviation is equal to the maximum vertical distance between the Lorenz curve and the diagonal of equality. This is represented by a dotted line in Figure A-1.

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